Math 250
Fall 2013
Midterm 2 - Form A
11/11/13

Name (Print): ____________________

This exam contains 6 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	10	
2	10	
3	10	
4	20	
5	20	
6	15	
7	15	
Total:	100	

1. (10 points) Find the inverse of the following matrix

$$A = \begin{bmatrix} 0 & 2 & -1 \\ 1 & -1 & 2 \\ 2 & -1 & 3 \end{bmatrix}$$

Ans:

$$A^{-1} = \begin{bmatrix} -1 & -5 & 3 \\ 1 & 2 & -1 \\ 1 & 4 & -2 \end{bmatrix}$$

2. (10 points) Find all values of c so that the following matrix is **not** invertible.

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & c \\ 0 & c & -15 \end{bmatrix}$$

Ans: Expanding along the first column: $Det(A) = 1(-45 - c^2) - 2(-30 + c) = -c^2 - 2c + 15 = (5 + c)(3 - c)$. A is not invertible iff $Det(A) = 0$ so $c = -5, 3$.

3. (10 points) Solve the following system using Cramer's rule:

$$\begin{cases} 2x_1 & -x_2 & +x3 & = -5 \\ x_1 & & -x_3 & = 2 \\ -x_1 & +3x_2 & +2x_3 & = 1 \end{cases}$$

Ans:

$$det\left(\begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 3 & 2 \end{bmatrix}\right) = 10.$$

$$det\left(\begin{bmatrix} -5 & -1 & 1 \\ 2 & 0 & -1 \\ -1 & 3 & 2 \end{bmatrix}\right) = -4.$$

$$det\left(\begin{bmatrix} 2 & -5 & 1 \\ 1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}\right) = 18.$$

$$det\left(\begin{bmatrix} 2 & -1 & -5 \\ 1 & 0 & 2 \\ -1 & 3 & 1 \end{bmatrix}\right) = -24.$$

So

$$
\begin{aligned}
x_1 &= \frac{-4}{10} = \frac{-2}{5} \\
x_2 &= \frac{18}{10} = \frac{9}{5} \\
x_3 &= \frac{-24}{10} = \frac{-12}{5}.
\end{aligned}
$$

4. Let

$$A = \begin{bmatrix} 1 & 2 & 4 \\ -1 & -1 & -1 \\ -1 & 0 & 2 \end{bmatrix}$$

(a) (5 points) Find the REF or RREF of A.
Ans:

$$R = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}.$$

(b) (5 points) Find a basis for Col A.
Ans:

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} \right\}.$$

(c) (5 points) Find a basis for Null A.
Ans:

$$x = \begin{bmatrix} 2x_3 \\ -3x_3 \\ x_3 \end{bmatrix}.$$

So a basis is

$$\left\{ \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} \right\}.$$

(d) (5 points) Find a basis for Row A.
Ans:

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} \right\}$$

5. A linear transformation $T : R^4 \to R^3$ is given:

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 & -2x_2 & +x_3 & +x_4 \\ 2x_1 & -5x_2 & +x_3 & +3x_4 \\ x_1 & -3x_2 & & +2x_4 \end{bmatrix}$$

(a) (5 points) Find the standard matrix A associated with T.

$$A = \begin{bmatrix} 1 & -2 & 1 & 1 \\ 2 & -5 & 1 & 3 \\ 1 & -3 & 0 & 2 \end{bmatrix}.$$

(b) (5 points) Find the REF or RREF of A.

$$R = \begin{bmatrix} 1 & -2 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

(c) (5 points) Find a basis for the range of T.

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ -5 \\ -3 \end{bmatrix} \right\}$$

(d) (5 points) Find a basis for the null space of T.

$$x = \begin{bmatrix} -3x_3 + x_4 \\ -x_3 + x_4 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} -3 \\ -1 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}.$$

So a basis is:

$$\left\{ \begin{bmatrix} -3 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

6. (15 points) Find all eigenvalues of

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 2 & 5 & 0 \\ 1 & -2 & -1 \end{bmatrix}$$

Ans:

$$A - \lambda I = \begin{bmatrix} -1-\lambda & 0 & 0 \\ 2 & 5-\lambda & 0 \\ 1 & -2 & -1-\lambda \end{bmatrix}$$

Expanding along the first row:

$$det(A - \lambda I) = (-1-\lambda)(5-\lambda)(-1-\lambda) = 0$$

iff $\lambda = 5, -1$.

7. (15 points) For each of the eigenvalue in problem 6, find a basis for the corresponding eigenspace.

For $\lambda = -1$:

$$A - (-1)I = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 6 & 0 \\ 1 & -2 & 0 \end{bmatrix}.$$

The REF is

$$R = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So a basis for eigeinspace is

$$\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

For $\lambda = 5$:

$$A - 5I = \begin{bmatrix} -6 & 0 & 0 \\ 2 & 0 & 0 \\ 1 & -2 & -6 \end{bmatrix}.$$

The REF is

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

So a basis for eigeinspace is

$$\left\{ \begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix} \right\}.$$